Quantum Computing and Linear Systems

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1. Introduction

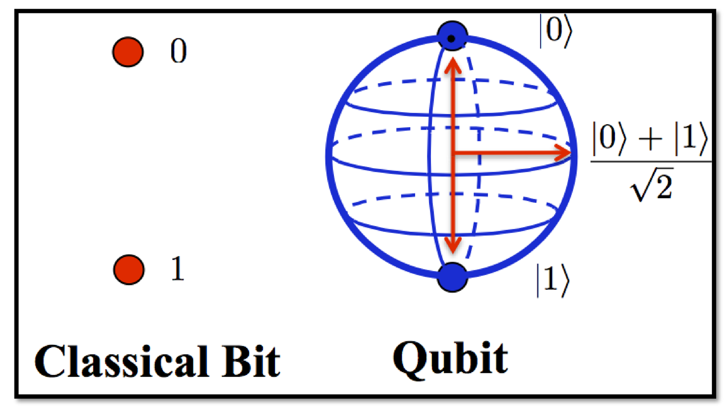
Recent investment from the private sector into quantum computing has led to several commercially available quantum computing devices. Among them the first publicly available quantum gate computer is the IBMQ. Alongside the hardware device is the open source software package QISKit which allows anyone to run a quantum circuit on the IBMQ machines.

There are many algorithms that offer a computational speedup over their best classical counterparts. One such algorithm was developed by Harrow, Hassidim, and Lloyd(HHL) in 2009 to solve linear systems of equations. The best classical algorithm, conjugate gradient scales as $O(N\log(N))$ while the HHL algorithm scales as $O(\log(N))$, where N is the size of the linear system.

Here we investigate the feasibility of implementing the HHL algorithm on the IBMQ. We begin with a study of the IBMQ focusing on the decoherence time. This is a measure of how long a qubit will remain in the state it is supposed to be in. Then we discuss the linear systems algorithm and its components. Results for the quantum Fourier transform(QFT) are presented and the improvements to the IBMQ required to run the HHL algorithm are discussed.

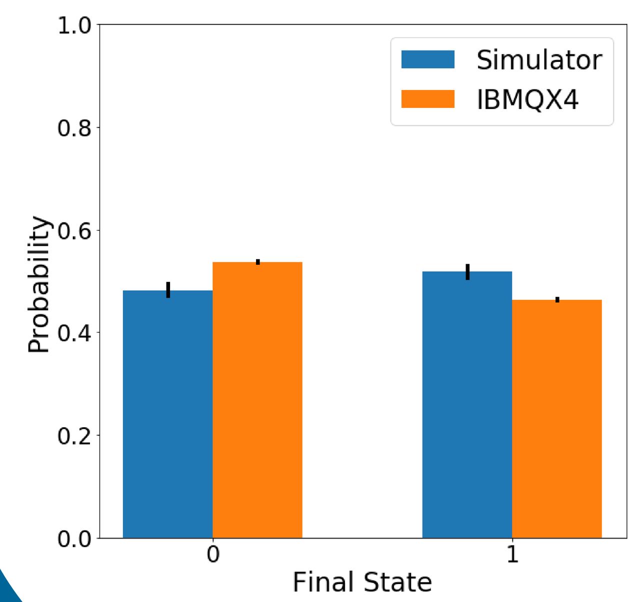
2. Quantum Computing

The fundamental unit of storage for quantum computers are two state quantum systems called qubits. Quantum mechanical states have the unique quality of being able to be in a superposition of states – a linear combination of all possible states. By performing many measurements on a qubit we obtain the probability it was in a specific state.

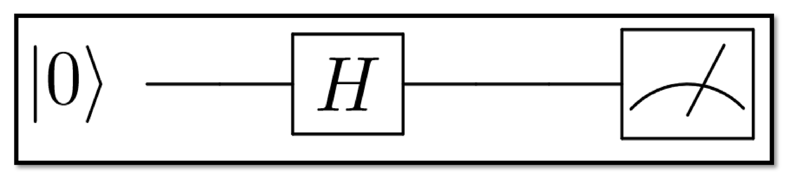


$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$

Operations can be applied to qubits to alter the state they are in. An example of an operation is the Hadamard gate H. Given a qubit in state $|0\rangle$, the H gate results in the qubit being in a superposition of $|0\rangle$ and $|1\rangle$. In quantum mechanics operators are represented as matrices and states as vectors. The H gate applied to a qubit in state $|0\rangle$ is shown on the left.

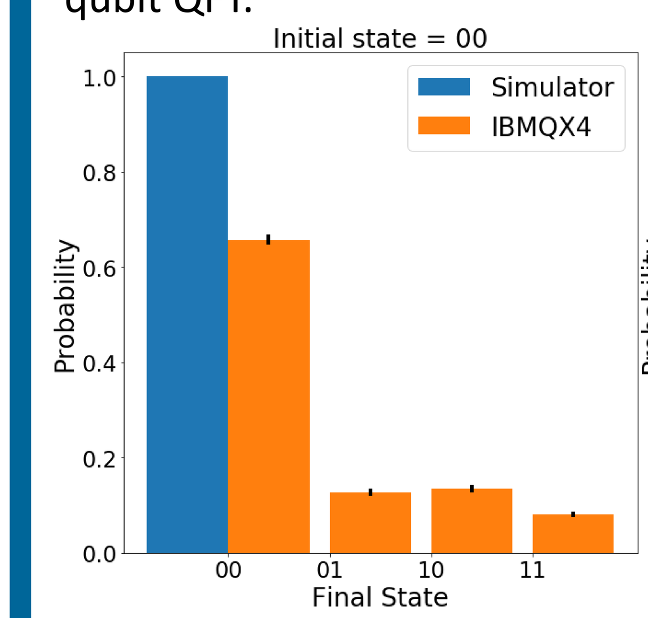


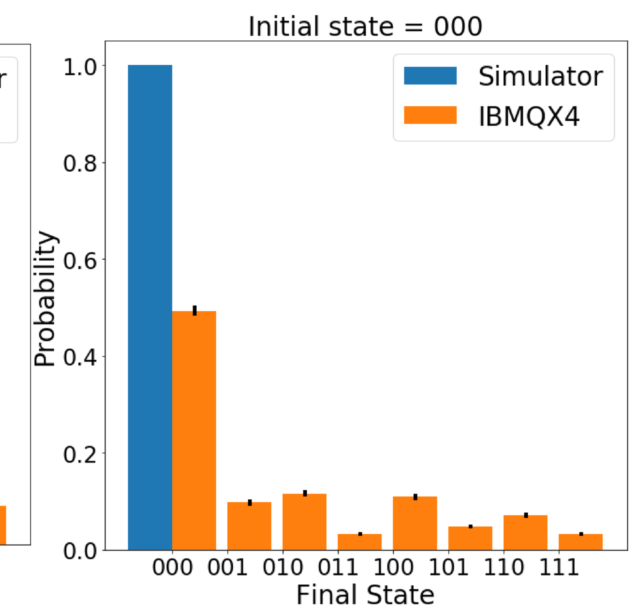
A collection of operations on a qubit is viewed as a circuit. These circuits can be simulated on a classical PC, or run on a real quantum device. The results of quantum circuits are probabilities the qubit was in a given state $|i\rangle$ where i is 0 or 1. The probability of it being in the state i is c_i^2 where c_i is the coefficient in front of the state $|i\rangle$. Below is a circuit implementing the operation of the H gate. On the left are the probabilities of the qubit being in $|0\rangle$ or $|1\rangle$ from a simulation and on the physical IBMQX4.



5. Current Results

To test the QFT we apply it and the inverse QFT on the initial states |00> and |000>. The QFT scales as $O(N^2)$ where N is the number of qubits that are transformed. Worse results are expected for the three qubit QFT. These experiments were performed on the IBMQX4 and are shown below. The left hand plot is the two qubit QFT, the right hand plot is the three qubit QFT.

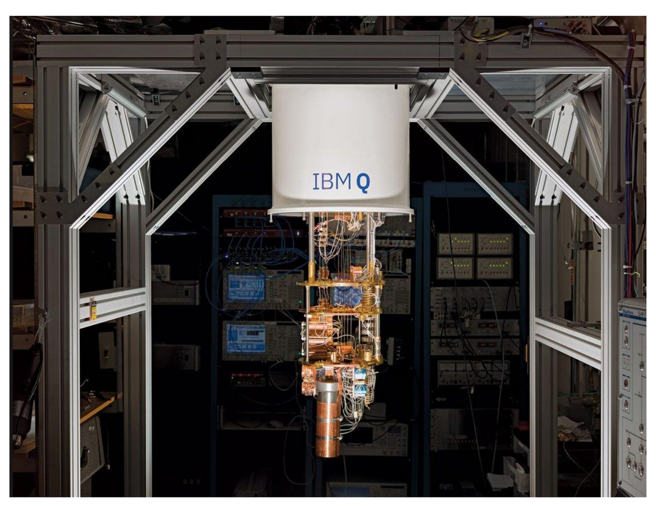




There is only a 50% probability we recover the initial state. The initial state was chosen such that no additional gates are added to the circuit for state preparation. The Fourier transform on three qubits only has approximately 50 gates which is well short of the decoherence time. Another large source of error in the IBMQ are two qubit gates. Of the 50 gates in the 3 qubit QFT, 20 of them were two qubit gates.

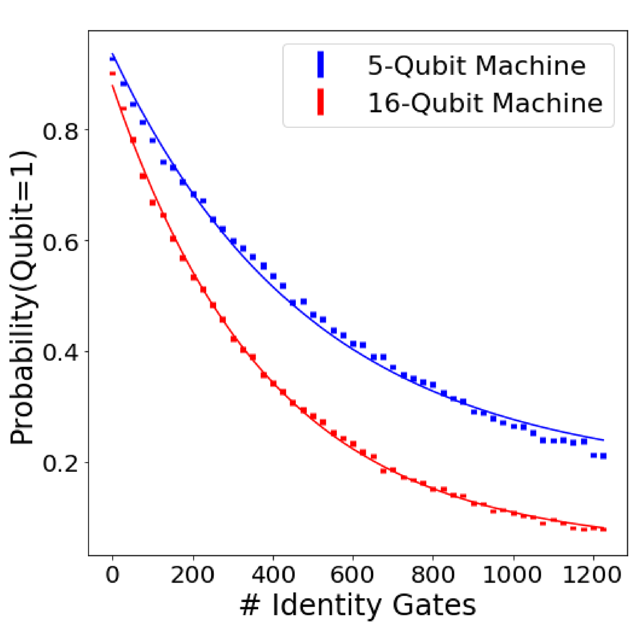
3. IBMQ

The work was tested experimentally on the IBMQ device. IBM currently has two functioning quantum gate computers with 5 and 16 qubits with plans on a 50 qubit machine in the near future. These machines can be run quantum circuits through QISKit.

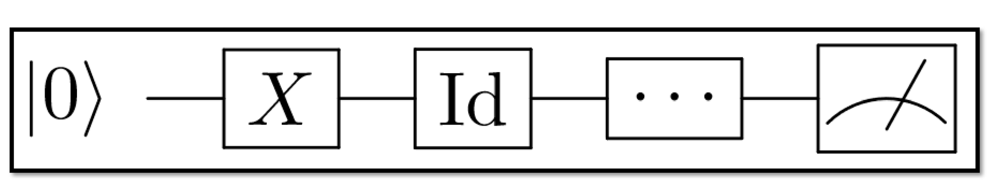


IBMQ

Quantum computing devices are still in their infancy and suffer from various types of decoherence. Decoherence is a loss of information about the quantum system's state to the environment. One example of this is energy relaxation. The qubit has two states $|0\rangle$ and $|1\rangle$. The state $|1\rangle$ is a higher energy state thus a qubit in state $|1\rangle$ will eventually decay into the lower energy state $|0\rangle$.



This shows the decoherence of a qubit on both IBMQ machines. The quantum circuit to test this is shown below. The X gate transforms the qubit from $|0\rangle$ to $|1\rangle$. Then we perform sequential identity operations, which do not effect he qubit, then measure the qubit.



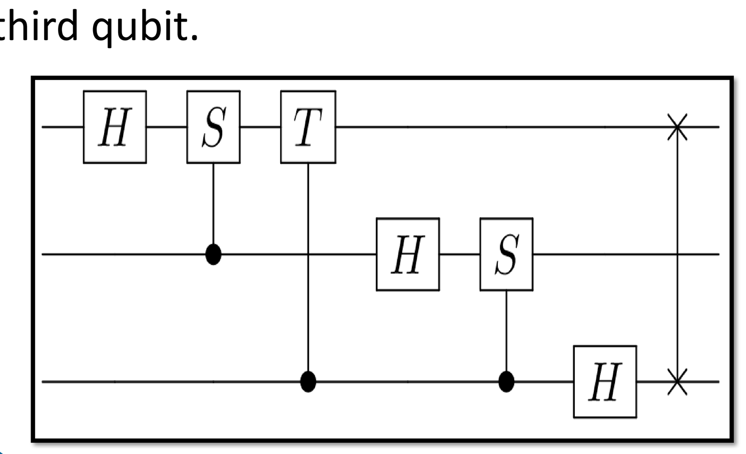
4. Solving Linear Systems

The algorithm for solving a linear system $A\vec{x}=\vec{b}$ for $\vec{x}$ on a quantum computer begins by putting the machine into the state $|0\rangle_a\,|b\rangle$. The label a is for an ancilla qubit which is used to select the correct final quantum state. The following operations are then performed to complete the HHL algorithm.

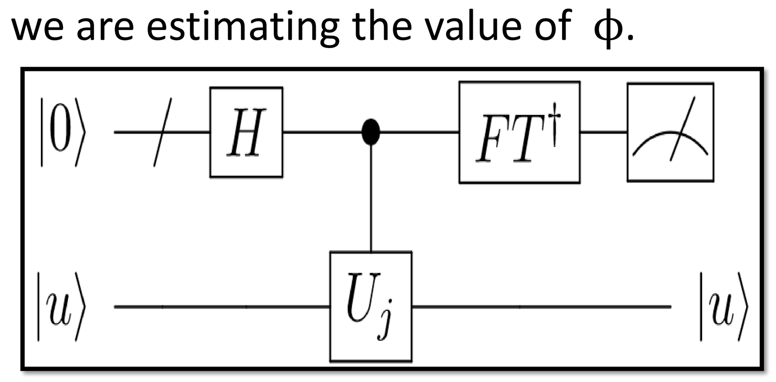
Prepare b → Phase Estimation → Controlled Rotation → Inverse Phase Estimation → Measure M

The final state of the quantum computer is $(|0\rangle + \frac{1}{\lambda}|1\rangle_a)|u\rangle$. Here u are the eigenvectors of A and λ are the eigenvalues. By selecting the results with a $|1\rangle$ in the ancilla bits the remaining qubits are left in an approximation of $\vec{x}$. The quantum computer is then used to measure expectation values of $|x\rangle$ with some operator M such as $\langle x|M|x\rangle$.

The quantum Fourier transform(QFT) is a component of phase estimation. Below is a circuit diagram for the QFT on 3 qubits. The controlled S and T operations only apply S to the target qubit if the control qubit is 1. S is the phase gate and T is the π/8 gate. The last gate is a swap gate swapping the first and third qubit.



The quantum phase estimation algorithm estimates the phase of an eigenvalue of a unitary operator U. This means we write an eigenvalue of U as $\exp(2\pi i\phi)$ and estimate the value of ϕ. Below is the circuit on p+a qubits. Here p is the number of qubits required for the eigenvector $|u\rangle$ of U and a is the number of qubits of accuracy with which we are estimating the value of ϕ.



6. Conclusion

The commerical interest in quantum computing is quickly advancing the size and performance of quantum devices. The IBMQ is the first commercially available quantum gate computer. The 5 and 16 qubit machines can be used to implement quantum circuits and a 50 qubit machine is on the way. The decoherence times can be measured so that the maximum circuit size in terms of single qubit gates can be estimated. Similar experiments need to be performed to better understand the errors associated with two qubit gates.

The HHL algorithm has several components, each of which need to perform well to give the full algorithm a chance at success. We analyzed the QFT and found that trying to QFT and inverse QFT a 3 qubit system has a 50% chance of success. This shows that the limiting factor on the IBMQ is not the number of qubits but the amount of gates that can be applied to the qubits. Success for linear systems solvers will rely on establishing longer decoherence times and having less errors on two qubit gates.

References

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